Instructions: Complete each of the following exercises for practice.

1. Compute the (signed) Jacobian of the transformation.
   (a) $(x, y) = (2u + v, 4u - v)$
   (b) $(x, y) = \left(u^2 + uv, uv^2\right)$
   (c) $(x, y) = (s\cos(t), t\cos(s))$
   (d) $(x, y) = (pe^q, qe^p)$

2. Use the given transformation to evaluate the integral $\iint_R f(x, y)\, dA$.
   (a) $f(x, y) = x - 3y$;
   R is the triangular region with vertices $(0, 0)$, $(2, 1)$, and $(1, 2)$;
   $(x, y) = (2u + v, u + 2v)$
   (b) $f(x, y) = 4x + 8y$;
   R is the parallelogram with vertices $(-1, 3)$, $(1, -3)$, $(3, -1)$, and $(1, 5)$;
   $(x, y) = \left(\frac{1}{4}(u + v), \frac{1}{4}(v - 3u)\right)$
   (c) $f(x, y) = x^2$;
   R is the region bounded by the ellipse $9x^2 + 4y^2 = 36$;
   $(x, y) = (2u, 3v)$
   (d) $f(x, y) = x^2 - xy + y^2$;
   R is the region bounded by the ellipse $x^2 - xy + y^2 = 2$;
   $(x, y) = \left(\sqrt{2}u - \sqrt{\frac{2}{3}}v, \sqrt{2}u + \sqrt{\frac{2}{3}}v\right)$
   (e) $f(x, y) = xy$;
   R is the region in the first quadrant bounded by $y = x$, $y = 3x$, $xy = 1$, and $xy = 3$;
   $(x, y) = \left(\frac{u}{v}, v\right)$
   (f) $f(x, y) = y^2$;
   R is the region bounded by $xy = 1$, $xy = 2$, $xy^2 = 1$, $xy^2 = 2$;
   $(u, v) = \left(xy, xy^2\right)$

3. Compute $\iint_R f(x, y)\, dA$ by making an appropriate change of variables.
   (a) $f(x, y) = \dfrac{x - 2y}{3x - y}$;
   R is the parallelogram given by $0 \leq x - 2y \leq 2$ and $1 \leq 3x - y \leq 8$
   (b) $f(x, y) = (x + y)\exp(x^2 - y^2)$;
   R is the rectangle given by $0 \leq x - y \leq 2$ and $0 \leq x + y \leq 3$
   (c) $f(x, y) = \cos\left(\dfrac{y - x}{x + y}\right)$;
   R is the trapezoidal region with vertices $(1, 0)$, $(2, 0)$, $(0, 2)$, and $(0, 1)$
   (d) $f(x, y) = \sin(9x^2 + 4y^2)$;
   R is the region in the first quadrant bounded by the ellipse $9x^2 + 4y^2 = 1$
   (e) $f(x, y) = \exp(x + y)$;
   R is the region satisfying inequality $|x| + |y| \leq 1$

4. Prove $\iint_R f(x + y)\, dA = \int_{uf(u)=u}^{0} 1\, du f(u)$ for every continuous function f on $[0, 1]$ and R the triangular region with vertices $(0, 0)$, $(1, 0)$, and $(0, 1)$.

5. Use a double integral to compute the area of the indicated region.
   (a) One loop of the rose $r(\theta) = \cos(3\theta)$
   (b) The region enclosed by both of the cardioids $r_1(\theta) = 1 + \cos(\theta)$ and $r_2(\theta) = 1 - \cos(\theta)$
   (c) The region inside the unit circle centered at $(1, 0)$ and outside the unit circle centered at the origin
   (d) The region inside the cardioid $r_1(\theta) = 1 + \cos(\theta)$ and outside the circle $r_2(\theta) = 3\cos(\theta)$

6. Compute the double integral $\iint_R f(x, y)\, dA$ for function $f(x, y)$ and region R.

(a) $\displaystyle\int_{x=0}^{2}\int_{y=0}^{\sqrt{4-x^2}} e^{-x^2-y^2}\ dy\ dx$

(b) $\displaystyle\int_{y=0}^{a}\int_{x=-\sqrt{a^2-y^2}}^{\sqrt{a^2-y^2}} (2x+y)\ dx\ dy$

(c) $\displaystyle\int_{y=0}^{\frac{1}{2}}\int_{x=\sqrt{3}y}^{\sqrt{1-y^2}} xy^2\ dx\ dy$

(d) $\displaystyle\int_{x=0}^{2}\int_{y=0}^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\ dy\ dx$